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Equilibrium

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# Optimal Resource Allocation in General Cournot-competitive Equilibrium

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## Abstract

Conventional economic theory stipulates that output in Cournot competition is too low relative to that which is attained in perfect competition. We revisit this result in a General Cournot-competitive Equilibrium model with two industries that differ only in terms of productivity. We show that in general equilibrium, the more efficient industry produces too little and the less efficient industry produces too much compared to an optimal scenario with perfect competition.

*Keywords:* Cournot oligopoly, GOLE (General Oligopolistic Equilibrium), industrial policy.

*JEL classification:* D50, H21, L13.

## 1 Introduction

Conventional economic theory should persuade us that industries operating under Cournot competition produce too little output. We demonstrate that this is not necessarily true in a model of general equilibrium. In a model with a fixed economy-wide input factor and two Cournot oligopolistic industries that differ only in terms of productivity, we show that the less efficient industry produces too much output, whereas the more efficient one produces too little. An optimal allocation of resources can be achieved by favouring the more efficient sector through appropriate transfers such as subsidies.

## 2 The Model

We use a closed-economy version of Neary's (2003; 2010) General Oligopolistic Equilibrium (GOLE) model. There is one closed economy with two industries,  $i = 1, 2$ , each producing one homogeneous good. In every

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industry there are  $n$  symmetric firms. Firms are relatively large in their own industries, and they have market power in their choice of output which is determined in a Cournot fashion. We assume, however, that each firm perceives itself to be small in the economy as a whole, and for this reason it treats economy-wide variables parametrically.<sup>1</sup>

We assume that entry is restricted such that firms make abnormal profits in equilibrium. All income accrues to the aggregate household, and labour is the only factor of production. The unit labour requirement, denoted  $\theta_i$ , differs across industries, and we assume that industry 1 is more efficient than industry 2. We normalise the unit labour requirement in sector 2,  $\theta_2 = 1$ , such that for sector 1 we have  $\theta_1 = \theta < 1$ . Preferences are represented by an additively separable utility function defined over the two goods, with each sub-utility function quadratic:

$$U = \sum_{i=1}^2 (aQ_i - \frac{1}{2}bQ_i^2), \quad (1)$$

where  $Q_i \equiv \sum_{j=1}^n q_{ij}$  is the total consumption of good  $i$  and  $q_{ij}$  is the output of firm  $j$  in industry  $i$ . Utility in (1) is maximised subject to the budget constraint,

$$\sum_{i=1}^2 P_i Q_i \leq I, \quad (2)$$

where  $P_i$  denotes the price of good  $i$  and  $I$  is aggregate income. The first-order condition for the consumer's optimisation problem gives the following inverse demand function for good  $i$ :

$$P_i = \frac{1}{\lambda} (a - bQ_i), \quad \text{with} \quad \lambda = \frac{a(\sum_{i=1}^2 P_i) - bI}{(\sum_{i=1}^2 P_i^2)}, \quad (3)$$

where  $\lambda$  is the marginal utility of income, that is, the Lagrange multiplier associated with the budget constraint. We denote the economy-wide wage rate in general equilibrium as  $w$ , and the government may implement an industry-specific subsidy denoted  $s_i$ . It is assumed that industrial policies are financed through lump-sum taxation of the representative household. Industrial policies and wage rates are determined endogenously, the manner in which is to be explained below. When setting Cournot outputs firms treat economy-wide variables  $\lambda$  and  $w$  as well as the policy variable  $s_i$  parametrically. Hence, we choose  $\lambda$  as our numeraire, such that every nominal variable is expressed in terms of the inverse marginal utility of income (i.e. real at the margin). We set  $\lambda = 1$ .

We begin by determining outputs. The profit of a firm  $j$  in industry  $i$  can be written as:

$$\pi_{ij} = (P_i - \theta_i w + s_i) q_{ij}. \quad (4)$$

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<sup>1</sup>We could also assume that each industry represents an aggregate mass of a continuum of industries. This would bring the model closer to the standard GOLE models, in addition to providing justification to the assumption that firms in each industry are “small” in the economy as a whole. Such extension, however, does not affect the qualitative results of the paper.

Maximising (4) with respect to the output of firm  $j$  using (3), we obtain the following equilibrium output of firm  $j$  in industry  $i$  and total output of industry  $i$  as:

$$q_{ij} = \frac{a - \theta_i w + s_i}{b(n+1)}, \quad \text{and} \quad Q_i = \frac{n(a - \theta_i w + s_i)}{b(n+1)}, \quad (5)$$

where, by symmetry,  $Q_i = nq_{ij}$ . The economy-wide wage rate can be solved from the full-employment condition of a fixed labour force  $L$ :

$$L = \theta Q_1 + Q_2 = \theta \frac{n(a - \theta w + s_1)}{b(n+1)} + \frac{n(a - w + s_2)}{b(n+1)}. \quad (6)$$

The equilibrium wage rate then becomes:

$$\bar{w} = \frac{na(1 + \theta) + n(\theta s_1 + s_2) - bL(n+1)}{n(1 + \theta^2)}, \quad (7)$$

where an overline is used to denote variables solved in general equilibrium. We assume that workers have an outside option yielding a zero surplus such that a participation constraint can be expressed as  $\bar{w} \geq 0$ . As is clear from (7), this implies a parameter restriction of  $a > \frac{bL(n+1)}{(1+\theta)n}$ , which also guarantees the existence of a Cournot equilibrium in the absence of industrial policy. Output in general equilibrium can now be derived as:

$$\bar{Q}_1 = \frac{na(1 - \theta) + n(s_1 - s_2\theta) + \theta bL(n+1)}{b(n+1)[\theta^2 + 1]}, \quad (8)$$

$$\bar{Q}_2 = \frac{na\theta(\theta - 1) + n\theta(s_2\theta - s_1) + bL(n+1)}{b(n+1)[\theta^2 + 1]}. \quad (9)$$

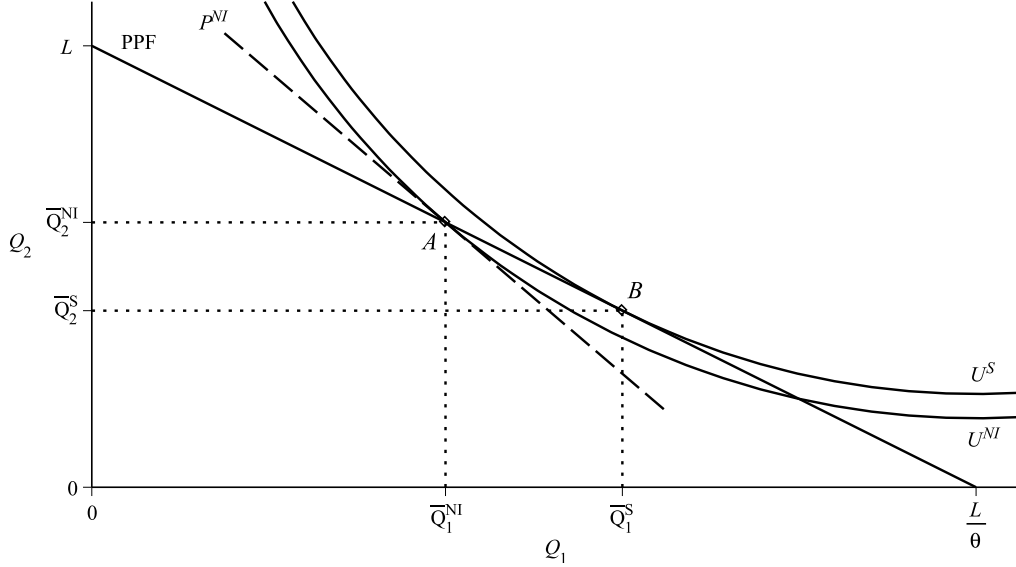
We first analyse the Cournot equilibrium in the absence industrial policies, that is  $s_1 = 0$  and  $s_2 = 0$ . This is done graphically in Figure 1. Equilibrium is attained at point  $A$ , and outputs in the two industries are denoted respectively,  $\bar{Q}_1^{NI}$  and  $\bar{Q}_2^{NI}$ , where “ $NI$ ” denotes “No Intervention”. The Production Possibilities Frontier (PPF) is given by the resource constraint in (6), which defines the feasible set. It is clear from the figure that the indifference curve passing through point  $A$  is not tangent to the economy’s PPF, but to the price line  $\bar{P}^{NI} = \frac{\bar{P}_1^{NI}}{\bar{P}_2^{NI}}$ . This leads to the following lemma:

**Lemma 1.** *The slope of the price line,  $\bar{P}^{NI}$ , which defines the marginal rate of substitution in the absence of industrial policy, is strictly less than the slope of the economy’s PPF if and only if  $\theta < 1$ .*

*Proof.* In Appendix. □

This implies that the representative consumer can achieve a higher utility within the feasible set by using industrial policy for  $\theta < 1$ . It is also clear from the Proof of Lemma 1 in the Appendix that if  $\theta = 1$ , the slope of the PPF is equal to  $P^{NI}$ , such that it is the productivity heterogeneity which causes the discrepancy. This is in line with Dixit and Grossman (1986), who find that the optimal industrial subsidy is zero in a model which does not feature productivity differences across industries. A government which is interested in

Figure 1: Equilibria in General Cournot-competitive Equilibrium.



maximising the utility of the representative consumer will set industrial policies to maximise indirect utility. We restrict industrial policy to industry 1,<sup>2</sup> and hence, the government solves the following problem:

$$\max_{s_1} G(\bar{Q}_1, \bar{Q}_2) \equiv U(\bar{Q}_1) + U(\bar{Q}_2) \quad \text{s.t.} \quad \bar{Q}_2 \geq 0 \quad \text{and} \quad s_2 = 0. \quad (10)$$

The constraint on  $\bar{Q}_2$  is there to ensure an interior solution. In the Appendix we show that the optimal industrial policy must satisfy:

$$s_1 = \begin{cases} \frac{a(1-\theta)}{n}, & \text{if } \frac{bL(n+1)}{(1+\theta)n} \leq a < \frac{bL}{\theta(1-\theta)}; \\ \frac{bL(n+1) - na\theta(1-\theta)}{n\theta}, & \text{if } \frac{bL}{\theta(1-\theta)} \leq a < \frac{bL(n+1)}{\theta(1-\theta)n}; \\ 0, & \text{if } a \geq \frac{bL(n+1)}{\theta(1-\theta)n}. \end{cases} \quad (11)$$

The unrestricted solution,  $\frac{a(1-\theta)}{n}$ , gives an interior solution for output which does not lie on the corner of the feasible set. Evaluating (8) and (9), respectively, at this subsidy yields:

$$\bar{Q}_1^S = \frac{a(1-\theta) + \theta bL}{b(\theta^2 + 1)}, \quad \text{and} \quad \bar{Q}_2^S = \frac{a\theta(\theta - 1) + bL}{b(\theta^2 + 1)}. \quad (12)$$

The new equilibrium  $B$  is plotted in Figure 1, where output of the more efficient industry 1 has increased and output of the less efficient industry 2 has decreased relative to the equilibrium without intervention. The indifference curve passing through point  $B$  is now tangent to the economy's PPF such that welfare

<sup>2</sup>We could also consider industrial policy for both industries, however, this would give us an infinite amount of solutions to pairs of subsidies and/or taxes guiding the economy towards its social optimum without yielding additional insight.

cannot be raised any further at that point. As the demand parameter  $a$  increases, the indifference curves become flatter such that the demand for variety relative to quantity decreases; as  $a$  exceeds  $\frac{bL}{\theta(1-\theta)}$ , the social optimum will thus be at the corner, that is,  $(\bar{Q}_1^S, \bar{Q}_2^S) = (\frac{L}{\theta}, 0)$ . In this case, the government implements a subsidy to industry 1 which drives the inefficient industry 2 out of business. For  $a \geq \frac{bL(n+1)}{\theta(1-\theta)n}$ , the Cournot equilibrium without intervention lies at the corner, in which case it is not possible to improve welfare within the feasible set, and the optimal subsidy is therefore zero. The reason for the suboptimal quantities without intervention is Cournot competition. In perfect competition where price equals marginal cost, the slopes of the price line and the PPF are equal, and equilibrium would be attained at point  $B$  without any need for industrial intervention. Interestingly, for the intermediate parameter range, industry 2 only exists due to Cournot competition; under perfect competition, this industry would not be able to stay active. This leads us to the following proposition:

**Proposition 1.** *In the present General Cournot-competitive Equilibrium model with two industries that differ only in terms of productivity, output of the more efficient industry is too low and that of the less efficient industry is too high relative to the social optimum. Optimal subsidies that satisfy equation (11) can be used to attain the optimal allocation of resources.*

*Proof.* In Appendix. □

Conventional economic theory suggests that Cournot quantities are too low relative to perfect competition. In this paper we have shown that this result does not necessarily hold in general equilibrium. In fact, the less efficient industry produces too much output relative to the more efficient one in general equilibrium. The intuition is that since the more efficient industry earns higher mark-ups and has relatively higher market power at the micro-level, then in a relative sense, it claims too little of the economy's fixed endowment of labour. Conventional industrial policy that uses micro-level welfare criteria to obtain an optimal subsidy for the less efficient industry leads to adverse welfare effects at the macro-level if the more efficient industry is not granted an even higher subsidy to compensate in general equilibrium.

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# Appendix

## Proof of Lemma 1

The slope of the economy's PPF can be found from (6) as  $-\theta$ . The marginal rate of substitution at the Cournot equilibrium in the absence of intervention,  $\bar{P}^{NI} = \frac{\bar{P}_1^{NI}}{\bar{P}_2^{NI}}$ , can be found by evaluating the inverse demand function (3) at, respectively,  $\bar{Q}_1$  and  $\bar{Q}_2$  from (8) and (9) for  $s_1 = 0$  and  $s_2 = 0$ , and then dividing the resulting expressions. We have,

$$\bar{P}^{NI} = \frac{a(n+1)[\theta^2 + 1] - na[1 - \theta] - \theta bL(n+1)}{a(n+1)[\theta^2 + 1] + na\theta[1 - \theta] - bL(n+1)},$$

such that  $-\bar{P}^{NI} < -\theta$  if and only if:

$$\frac{a[\theta^2 + 1][1 - \theta]}{a(n+1)[\theta^2 + 1] + na\theta[1 - \theta] - bL(n+1)} > 0.$$

This equation is satisfied if and only if  $\theta < 1$ , but notice also that for  $\theta = 1$ , the slopes must be equal. **q.e.d.**

## Derivation of Eq. (11)

Using the equilibrium quantities in, respectively, (8) and (9), the Lagrangian for the maximisation problem in (10), takes following form:

$$\mathcal{L} = \left( a\bar{Q}_1 - \frac{b}{2}\bar{Q}_1^2 \right) + \left( a\bar{Q}_2 - \frac{b}{2}\bar{Q}_2^2 \right) + \mu\bar{Q}_2,$$

where  $\mu$  is the Lagrange multiplier for the constraint  $\bar{Q}_2 \geq 0$ . The Kuhn-Tucker conditions for this maximisation problem are:

$$\begin{aligned} \frac{d\mathcal{L}}{ds_1} &= \frac{(a(1-\theta) - ns_1)n}{b(n+1)^2[\theta^2 + 1]} - \mu \frac{n\theta}{b(n+1)[\theta^2 + 1]} = 0; \\ \frac{\mathcal{L}}{d\mu} &= \mu \left( \frac{na\theta(\theta - 1) - n\theta s_1 + bL(n+1)}{b(n+1)[\theta^2 + 1]} \right) \geq 0, \text{ with } \mu = 0 \text{ if } na\theta(\theta - 1) - n\theta s_1 + bL(n+1) > 0, \end{aligned}$$

where the constraint  $s_2 = 0$  has been used. The unconstrained solution, where  $\mu = 0$ , can be found from the first condition as:  $s_1^{UC} = \frac{a(1-\theta)}{n}$ , and the parameter range for which this solution is valid can be found by substituting it into  $\bar{Q}_2$  in (9):

$$\bar{Q}_2(s_1^{UC}) > 0 \Rightarrow \frac{a\theta(\theta - 1) + bL}{b(\theta^2 + 1)} > 0 \Rightarrow a < \frac{bL}{\theta(1 - \theta)}.$$

The constrained solution for  $s_1$  can be found from the second Kuhn-Tucker condition, and together with the first condition, we have:

$$s_1 = \frac{bL(n+1) - na\theta(1 - \theta)}{n\theta}, \quad \text{and} \quad \mu = \frac{bL}{n\theta^2}.$$



It is also possible that the Cournot equilibrium lies at the corner of the feasible set, that is  $(\bar{Q}_1^{NI}, \bar{Q}_2^{NI}) = (\frac{L}{\theta}, 0)$ . To rule this out we need:

$$\bar{Q}_2^{NI} > 0 \Rightarrow \frac{na\theta(\theta - 1) + bL(n + 1)}{b(n + 1)[\theta^2 + 1]} > 0 \Rightarrow a < \frac{bL(n + 1)}{\theta(1 - \theta)n}.$$

## Proof of Proposition 1

Denote by  $\bar{P}_1^S$  and  $\bar{P}_2^S$ , respectively, the prices faced by the representative consumer when optimal policies in (11) are implemented. The slope of  $\bar{P}^S = \frac{\bar{P}_1^S}{\bar{P}_2^S}$ , or the marginal rate of substitution with optimal subsidies, can be found by evaluating the inverse demand function (3) at, respectively,  $\bar{Q}_1^S$  and  $\bar{Q}_2^S$ , from (12), and then dividing the resulting expressions. We have,

$$-\bar{P}^S = -\theta,$$

which is also the slope of the PPF in (6). For the corner solution where  $(\bar{Q}_1, \bar{Q}_2) = (\frac{L}{\theta}, 0)$ , the marginal rate of substitution is  $\frac{a-b\frac{L}{\theta}}{a}$ . This is greater than the slope of the PPF,  $-\theta$ , if and only if  $a > \frac{bL}{\theta(1-\theta)}$ , which is satisfied at the corner. **q.e.d.**

## References

- Dixit, A. K. and Grossman, G. M. (1986). Targeted export promotion with several oligopolistic industries. *Journal of International Economics*, 21(3-4):233–249.
- Neary, J. P. (2003). Competitive versus comparative advantage. *The World Economy*, 26(4):457–470.
- Neary, J. P. (2010). Two and a half theories of trade. *The World Economy*, 33(1):1–19.